THE MYSTERY OF FORMICA'S TRIANGLES

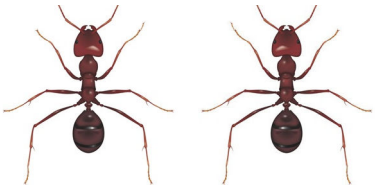
by Tom Peachey

With apologies to Edwin Abbott, W. W. Sawyer and Frank Dickens

1 Formica the Flant

Once there was a world who's name is Forgotten.[1] The highest lifeform there were some ants, but not just any ants. These ants were completely flat. And I mean completely - zero thickness. That was okay because their world was completely flat too. These ants could move forward and back, and side to side, but not up and down - there was no up and down. They could not even imagine up or down.

I forgot to tell you, the ants are called *flants* in their language, *Flantspeak*. The picture here shows what they look like.



Top view **Bottom view** **Side view**

Despite their lack of thickness, flants are quite intelligent and live civilised and interesting lives. The hero of our story is a flant named Formica. He works as a surveyor. But not just any surveyor, Formica is fifth in line for Surveyor General. Currently he is in charge of a complete remapping of the Queendom. This is his big chance to get promotion to fourth in line.

At home Formica lives with his wife Mica. I should explain, when a male flant marries he takes the name of his wife, just adding the prefix *For*. They have no children yet, although they apply each year for an egg from the Royal Eggbank, but without success so far with the coin flip.

Mica works as a philosopher, trained in this by her maiden aunts, Saken and Skin. Mica's work involves sitting and thinking deep thoughts. In Formica's eyes this is not real work. He is a practical flant, scurrying about the Queendom looking useful. This year he was even chosen to measure the foot of Queen Titude CCCXIV. But Formica pretends to be interested in his wife's work. That work seems to be thinking about impossible questions, such as why time only runs forward, and why is the universe just two-dimensional. Currently she is worrying whether the universe is infinite or bounded. An infinite universe just goes on forever which seems a waste. But with a finite universe, when you come to the end, there must be something beyond?

Long ago the Queendom had kings instead of queens. But they were brought up spoiled and reigned as gluttonous, cruel and dangerous. The "glorious revolution" replaced kings with queens but they turned out to be gluttonous, cruel and dangerous. Then it was decided to appoint queens at birth. Each would reign until the age of 10 when they would be replaced - before they got too dangerous. But the child queens rarely served a full term, dying from a surfeit

[1] This name means "attached to Gotten" But no-one can remember what Gotten is.

of sugar. That's why there is now an "aleatory monarchy"; all decisions are still made by the queen, but only by answering her advisor's true/false questions by flipping the royal coin.[2]

Despite having no real power, queens are widely adored, especially the cute ones. For example, the basic unit of length, the foot, is still taken as the length of the Queen's foot. This is measured each year on the Queen's birthday, and all recorded measurements are then adjusted to the new standard. This is one reason why Formica is so busy; each year he needs to convert all existing records.

2 Mica is not Convinced

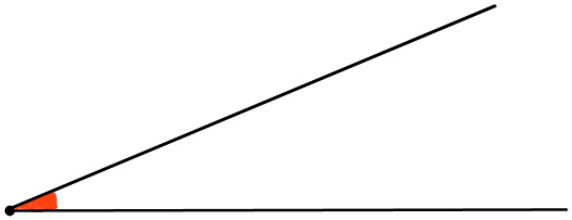
As we meet Formica he has just completed multiplying all distances in the database by the factor 0.949, and he has time to prepare for the Great Survey. Today he gets home tired and worried.
"You're worried" says Mica, "what's ahead?" [3]
"You know I'll be starting the Great Survey soon ..."
"Yes leaving me alone with no husband and no children. Did you know that Tunate next door has had a new baby every year for 12 years running. What are the chances of that?"
"One in 4096" Formica said automatically. '"'Anyway, Forgerri and I have been testing the new violet laser prtrctrs. They are meant to be incredibly accurate and are very expensive. But it looks like they're faulty".
"So take them back to the shop."
"Not so easy. They are imported from the Kngdm f Md."
"Md" exclaimed Mica, "they are so backward".
"Yes but ..."
"and dirty!"
"Sometimes muddy ..."
"They still have kings!"
"But ..."
"And they haven't invented vowels yet!"
"Yes but they are famous for the accuracy of their instruments. That's why we paid big money for their prtrctrs."
Mica was starting to see the problem. "What do these prtrctrs actually do?" she asked.
"They measure angles."
"And you want to measure angles because?..."
Formica was happy to talk about his important work. "This Great Survey will make a grid of triangles across the Queendom. We need to measure the triangles accurately."
"Why?"

[2] Not actually flipping, it's more like spinning.
[3] *What's ahead* is Flantspeak for *what's up*.

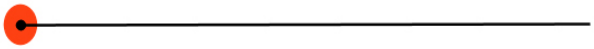
"Well, for example - so people will know exactly which foot of land belongs to which farmer. We measure lengths of our starting triangle and for all the other triangles we just measure angles and work out the sides using trigonometry"
"So ... these prtrctrs are not working?"
"They are working, but not accurately. Forgerri and I measured some large triangles, and the angles did not add up to 180 degrees. Always a bit greater."
"Do you want them to add to 180 degrees?"
"Of course. According to mathematics they *must* add to 180."
"Oh mathematics. I missed school the week they spent on that."
Formica made a smug smile. One week to learn mathematics! *He* went to Her Majesty's Special School for Very Smart Flants where they learned all mathematics in a day.
"But" continued Mica, "Aunty Saken did teach me algebra. Show me why the angles must add to 180. What is an angle anyway?"

Formica took out some chalk and drew a line segment. "This is a straight line."

"What do you mean 'straight'? "
Mica was in philosophy mode, but Formica was up for the challenge.
"It's the shortest path between two points"
"Um ..."
"And it's the path that light takes."
"Go on."
"Let's rotate this line about one end." He drew the new line.

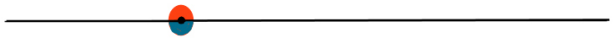


"This makes an angle - and the measure of the angle tells you how much it has rotated."
"Okay"
"If it rotates all the way back to the start, that is a rotation of 360 degrees."



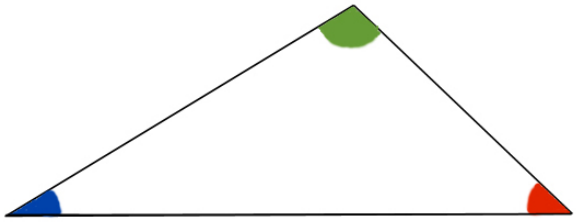
"Why 360?"
"Well the ancient Babyloniants used 360."
Mica was about to challenge, but Formica continued quickly.
"And our Queen has decreed it."

That always won the argument. Mica just nodded. Formica continued.
"So if we just rotate the line half way, we get an angle of 180 degrees on each side."

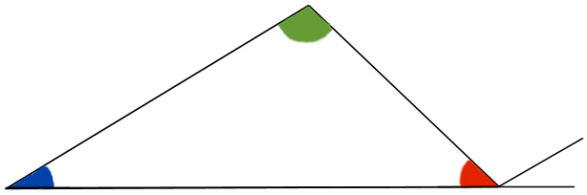


Mica nodded again; she was getting to understand this angle stuff.

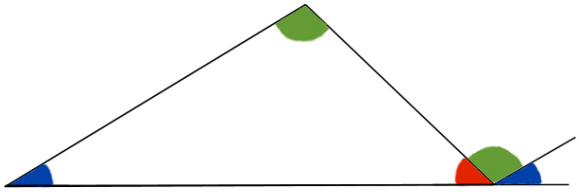
Formica was now sounding pompous. "I will now prove that the angles in a triangle add to 180 degrees."
"What is this *prove*?"
"A proof shows that something must be true, and shows why it is true."
Mica nodded.
'Now a *tri*angle is made of three straight lines. Making three angles."
He drew the triangle and coloured the angles.



"I want to show that these angles add to 180 degrees.
"Extend one side"
"And at the red corner draw a line parallel to the opposite side."



He was colouring the new angles to show that they added to 180,



but Mica interrupted. "What is a parallel line?"
Formica was unsure of this but tried to sound confident.
"Parallel lines never meet."
"How do you know they never meet?"

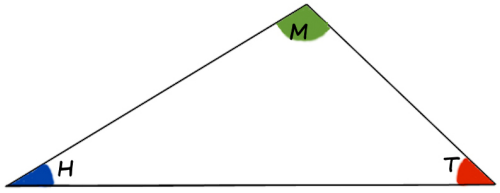
"Well, you walk along them and check."
"So if they are parallel, you could walk forever and never decide."
Formica stroked his antennas for a while, then stomped to his shed and started hammering some nails.

3 A Day Out

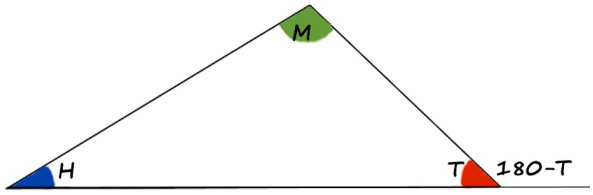
The next day was Full Moon Day. No work was done throughout the Queendom. The flants in each village would gather at the local temple to chant prayers. Then each family would walk three times around the temple before adjourning to tend the graves of their ancestors. Mica found this difficult - the other families each had their troop of excited children. She could feel the pity of other flant mothers for her barren family. Or worse. An absence of children was widely believed to be punishment for bad deeds in a previous life. So she was happy when it was time to leave.

Formica was not so happy. The next stop was a feast at his Mother-In-Law's nest. There, Mica's mother would quiz him about the chances of grandchildren and discuss at length his shortcomings as a For. But today Formica was not concentrating on his own inadequacies. His mind kept wandering to triangles. Walking home completed a triangle - from home to temple to Mother-In-Law to home. By the time he turned into home he had an idea.
"I have a new proof for the angles of a triangle." said Formica.
"Does it have parallel lines" replied an amused Mica.
"No. Just walking, and some algebra."
"Good. I like algebra."

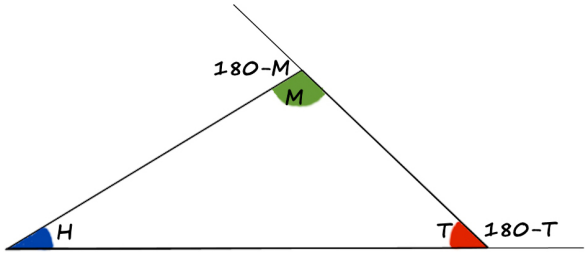
Formica drew a picture. "This point is H our home."
"Yes."
"And T is the temple – And M is your mother's place."
"Okay."
"So we have a triangle." He drew the triangle and coloured the three angles.



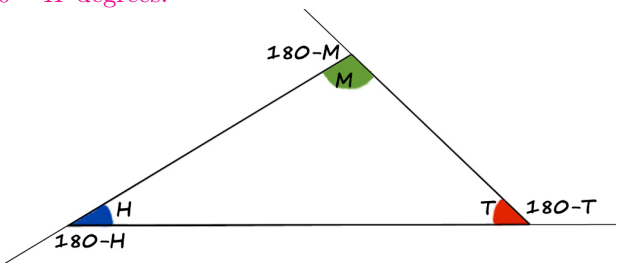
"Here T stands for the place of the temple. But I will also use T for the measure of the red angle."
"And the same for M and H?"
"Yes. I want to show that $T + M + H = 180$"
"Now suppose we walk from H to T. Then we turn toward M. We need to turn through an angle of $180 - T$ degrees."



Mica inspected the picture. "Yes that looks correct."
"At M we turn to walk home. This time we turn through an angle of $180 - M$ degrees."



"Good again."
"At home we turn again to face the temple."
"This time $180 - H$ degrees."



"Yes you get it. By this stage we have turned a complete circle, 360 degrees."
Mica was excited. "Let me solve this." And she wrote:

$$(180-T) + (180-M) + (180-H) = 360$$
$$540 - T - M - H = 360$$
$$540 - 360 = T + M + H$$
$$T + M + H = 180$$

"So ... they add up to 180"
Formica reached over and added the standard way to finish a proof.

$$T + M + H = 180$$
$$T.\ Y.\ S.$$

"Told you so." he said.
"Let me think" said Mica, "The angles must add to 180. But when you measure them they do not."
"Correct."
"It's as if the straight lines are bent."
Formica shook his head. "Straight means straight. Bent means bent".

4 A Flash of Inspiration

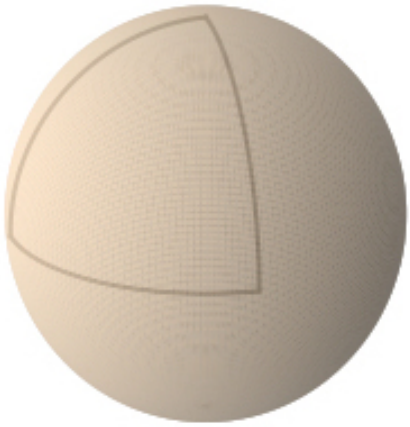
At work the next day Formica had this brain itch. Is straight really straight?
"Hey Forgerri! I have a new way to test the prtrctrs. No triangles!"
"How?"
"We put the angles together - like this." And he drew a picture with three angles.

"Easy" said Forgerri, "no walking needed."
And they set up a prtrctr and measured three angles like in the picture. Forgerri did the adding up.
"Exactly 180" he said "- or rather within 1 second of arc, highly accurate. What is going on? When the angles are together they make 180 degrees, but in a triangle they add to more."
Formica did not reply, he was transfixed, staring into space. Thinking.
"You should turn off the laser", said Forgerri.
Formica looked across. "It's not dangerous. If the beam hits a flant then they get a tickle and move out of the way." He was amused. "As we speak, the laser

might be tickling flants in Md – maybe even further.
Just then Formica felt someone tickling his tail. He turned, but there was no-one there. Then he saw the light. It was violet.

That was when Formica discovered how the universe works. Did he kiss Forgerri and do a little dance? Did he shout "Eureka" and run naked in the town? No, Formica was a serious flant, fifth in line for Surveyor General. He did however recite a little poem remembered from his schoolday when they learnt all poetry.

> "Then felt I like some antiMd official
> Who starts a war o'er something superficial,
> Or, like a flea that meets some other fleas,
> Silent, upon a Pekinese."



Mica and team measure a *huge* triangle

People have been asking what happened with Mica and Formica. Well, Mica joined the team and went on the Great Survey. While on the road, Mica and Formica developed a new type of trigonometry that works for bent straight lines. It turned out that they were an effective collaboration; Formica would bubble with ideas and Mica would shoot them down, except when she couldn't. They have become quite famous. The Queen has bestowed a family name – they are now Family Na-Pier.[4] Not that the have time to enjoy their fame. Their life is dominated by baby triplets, three little girl flants: Getful, Give and Eigner. Formica says it must have been a triple-yolker, but Mica disputes this. She claims that someone slipped an extra child into the cot. And she darkly adds "Be careful what you wish for".

Some readers might be wondering why I have told this story about these flat creatures in this flat world. I'm thinking it might be relevant to a problem in our real 3D universe. As you know, we have at last got messages from our colonies on Alpha Centauri and Lalande 21185. They complete a triangle with our Sun and we now have measurements of the triangle angles. It appears that the angles do not add to 180 degrees, in fact the total is slightly *less*. Go figure.

[4] *Na-Pier* is Scots Flantspeak for *No Equal.*